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The line shape of the Z boson [1]

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Abstract

At LEP 1, cross sections and cross section asymmetries may be analysed model independently. Cross sections depend on four, asymmetries on two free parameters. As an example, I discuss the model independent Z boson mass determination from the Z line shape and compare it to the Standard Model approach.

1 Introduction

The precision measurements of the weak neutral current reaction

$$e^+e^- \to (\gamma, Z) \to f^+f^-(n\gamma) \tag{1}$$

are being performed at LEP 1 and SLC since 1989. Until summer 1994, the following event samples have been collected [1]:

- $e^+e^- \to \bar{q}q$: 7.1 Mio. (LEP 1)
- $e^+e^- \to \bar{l}l$: 0.8 Mio. (LEP 1)
- $e^+e^- \to \mathrm{all}^3$: 0.05 Mio. (SLC)

From these data, one may derive the weak neutral current parameters with unprecendented precision. An unbiased interpretation of data becomes a highly nontrivial task since data have to be understood as a result not only of e^+e^- scattering with one particle exchange but also of

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[3] Excluding Bhabha events.

radiative corrections. The calculation of the latter may be, to some approximation, subdivided into two different, separated problems: the well understood and calculable QED and QCD corrections (including real bremsstrahlung of photons and gluons and higher order corrections) at one hand and, at the other, the model dependent virtual weak corrections. As long as the latter are small and may be absorbed into a small number of parameters it will be reasonable to hope to interprete data correctly without fixing the model. In fact, there are three different popular approaches to the Z line shape data:

- Standard Model ($\mathcal{SM}$): $\alpha_{em}, \alpha_{strong}, G_\mu, M_Z, m_t, M_H$;
- $\mathcal{SM}$ plus New Physics ($\mathcal{NP}$): assume the $\mathcal{SM}$ parameters as being known and determine the additional ones;
- Model independent ($\mathcal{MI}$): M_Z, Γ_Z and few others; see below.

An updated discussion of the $\mathcal{SM}$ approach is being prepared in [2]. In the following I will concentrate on the model independent approach to the Z line shape and the problem of a unique choice of parameters. An especially interesting question is that of the minimal number of free parameters.

The QED and QCD corrections may be taken into account with the following convolution formula:

$$\sigma(s) \;=\; \int \frac{ds'}{s}\sigma_0(s')\,\rho\left(\frac{s'}{s}\right) \;+\; \int \frac{ds'}{s}\sigma_0^{int}(s,s')\,\rho^{int}\left(\frac{s'}{s}\right), \tag{2}$$

where the radiator ρ describes initial and final state radiation, including leading higher order effects and soft photon exponentiation, while the second radiator ρ^{int} takes into account the initial-final state interference effects (see e.g. [3]), which are comparatively small (a few per mille) but maybe not negligible in future. The bulk of the QED corrections is absorbed in ρ, which is described in detail at many places, e.g. in [2, 3, 4] and in references therein. Aiming at an accuracy of per mille around the Z peak, the radiator ρ is different for cross sections, which are symmetric (like σ^T) or anti-symmetric (like σ^{FB}) in the scattering angle (see e.g. [3]; the same holds true for ρ^{int}). The QCD corrections (if any) are traditionally included as factors to the basic, elementary cross section σ_0; see e.g. [2].

If one considers the contribution from the initial-final state interferences to be negligible (or prefers to calculate them in the $\mathcal{SM}$), then the only unknown is the basic cross section as a function of the invariant s. Thus, the line shape problem has been reduced to the search for an ansatz for $\sigma_0(s)$. Under certain, weak assumptions one may e.g. derive from the data the following five parameters from which the cross sections may be constructed [1]:

$$\begin{aligned}
M_Z &= 91.188\,8 \pm 0.004\,4 \text{ GeV}, \\
\Gamma_Z &= 2.497\,4 \pm 0.003\,8 \text{ GeV}, \\
\sigma_0^{had} &= 41.49 \pm 0.12\,\text{nb}, \\
R_l = \frac{\sigma_0^{had}}{\sigma_0^{lept}} &= 20.795 \pm 0.040, \\
A_{FB,0}^{lept} &= 0.017\,0 \pm 0.001\,6.
\end{aligned} \tag{3}$$

These parameters are considered to be primary parameters in contrast to derived ones, e.g. the effective leptonic weak neutral current couplings or the effective weak mixing angle [1] (for

details see, again, [2]):

$$
\begin{aligned}
(g_v^l)^2 &= 0.001\,44 \pm 0.000\,14, \\
(g_a^l)^2 &= 0.251\,18 \pm 0.000\,56, \\
\sin^2 \vartheta_W^{eff} \equiv \frac{1}{4}\left(1 - \frac{g_v^l}{g_a^l}\right) &= 0.231\,07 \pm 0.000\,90.
\end{aligned} \tag{4}
$$

Another interesting derived quantity is the invisible width of the Z boson, which may be derived from Γ_Z and the observed partial Z widths; or, alternatively, the number of light neutrino species [1]:

$$N_\nu = 2.988 \pm 0.023. \tag{5}$$

2 Model independent determination of the Z mass

The Z boson mass determination is part of a global fit to a large variety of Z line shape data. It is dominated by the total hadronic production cross section due to the high statistics of that reaction. A sufficiently accurate ansatz for this cross section is [5, 6, 7, 8]:

$$\sigma_0(s) = \frac{4}{3}\pi\alpha^2 \left[\frac{r^\gamma}{s} + \frac{s \cdot r + (s - M_Z^2) \cdot j}{|s - M_Z^2 + is\Gamma_z/M_Z|^2}\right], \tag{6}$$

where the photon exchange parameter r^γ is assumed to be known. The numerical value of the Z mass is closely related to the peak position $\sqrt{s_{\max}}$ of the Z line shape; thus an estimate of the peak position models problems connected with the mass measurement. As a simplified ansatz let us use the following approximation of the Breit-Wigner function:

$$\sigma_0^Z(s) \sim \frac{M_Z^2 \cdot r}{|s - M_Z^2 + iM_Z\Gamma_Z|^2}. \tag{7}$$

The bulk of the corrections is due to initial state radiation and may be well described by the following formula [9] (see also [3] and the discussion in [4]):

$$
\begin{aligned}
\rho(z) &= \beta(1-z)^{\beta-1}\delta^{S+V} + \delta_1^H + \delta_B^H, && (8)\\
\beta &= \frac{2\alpha}{\pi}(L-1), \quad \text{with} \quad L = \ln\frac{s}{m_e^2}, && (9)\\
\delta^{S+V} &= 1 + \frac{\alpha}{\pi}\left[\frac{3}{2}L + 2\zeta(2) - 2\right] + \left(\frac{\alpha}{\pi}\right)^2 \left\{\left[\frac{9}{8} - 2\zeta(2)\right]L^2 + \left[-\frac{45}{16} + \frac{11}{2}\zeta(2) + 3\zeta(3)\right]L \right.\\
&\quad \left. + \left[-\frac{6}{5}\zeta(2)^2 - \frac{9}{2}\zeta(3) - 6\zeta(2)\ln 2 + \frac{3}{8}\zeta(2) + \frac{19}{4}\right]\right\}, && (10)\\
\delta_1^H &= -\frac{\alpha}{\pi}(1+z)(L-1), && (11)\\
\delta_B^H &= \left(\frac{\alpha}{\pi}\right)^2 \frac{1}{2}(l-1)^2 \left\{(1+z)\left[3\ln z - 4\ln(1-z)\right] - \frac{4}{1-z}\ln z - 5 - z\right\}, && (12)
\end{aligned}
$$

with $z = s'/s$. The corresponding peak shift, which has to be taken into account in order to correctly determine the Z mass is known since long [10]:

$$\sqrt{s_{\max}} - M_Z \;=\; \delta_{QED} = \frac{\pi}{8}\beta\left(1 + \delta^{S+V}\right)\Gamma_Z + \text{small corrections} \;\approx\; 90\,\text{MeV}, \tag{13}$$

where δ^{S+V} describes virtual and soft real photon emission.

Improving the Breit-Wigner function by a replacement of $M_Z^2 \cdot r$ by $s \cdot r$ and of the Z width in the denominator by an energy dependent width function as preferred by the LEP collaborations, the peak shift gets modified as follows:

$$\sqrt{s_{\max}} - M_Z \;=\; \delta_{QED} \oplus \frac{1}{4}\frac{\Gamma_Z^2}{M_Z} \ominus \frac{1}{2}\frac{\Gamma_Z^2}{M_Z}. \tag{14}$$

If these shifts are neglected, they have to be condered as systematic errors. The importance of the second of the modifications has been stressed first in [11, 12]. It amounts to $-\Gamma_Z^2/(2M_Z) = -34$ MeV.

The influence of the γZ interference on the Z mass determination has been observed some time ago; see e.g. [5]. It leads to an additional modification of the the Z peak position:

$$\sqrt{s_{\max}} - M_Z \;=\; \delta_{QED} \oplus \frac{1}{4}\frac{\Gamma_Z^2}{M_Z}\left(1+\frac{j}{r}\right) \ominus \frac{1}{2}\frac{\Gamma_Z^2}{M_Z}. \tag{15}$$

The interesting point is the following. Neglecting this interference (setting j=0) leads to an erraneous systematic shift of the Z mass of 17 MeV$\otimes(j/r)$. If one wants to take into account the j, a model for its prediction is needed.

The Z line shape has four free parameters per channel: M_Z, Γ_Z, r, j. If j is not fixed but treated as a parameter of a $\mathcal{MI}$ line shape fit, the Z mass gets an additional error of $\Delta M_Z = \pm 8$ MeV/expt. This uncertainty could be removed by a dedicated running of LEP 1 at energies away from the peak [13, 14].

3 $\mathcal{MI}$ approach to asymmetries

For the description of cross section asymmetries, an analogue to (2) may be used [7]:

$$\mathcal{A}(s) = \frac{\sigma_A(s)}{\sigma_T(s)}, \tag{16}$$

where subscript A stands for the type of asymmetry, e.g. $A = FB$ for the forward backward asymmetry, and both cross sections σ_0^A and σ_0^T have the form (6), with different parameters r^A, j^A. Around the Z resonance all asymmetries take an extremely simple form:

$$\bar{\mathcal{A}}_A(s) \;=\; \bar{A}_0^A + \bar{A}_1^A\left(\frac{s}{M_Z^2} - 1\right), \tag{17}$$

where the coefficients at the right hand side depend on r^A, j^A and on the QED corrections. In [5, 7] it is explained in detail that the coefficient $\bar{A}_0^A \approx r^A/r^T$ is for $A = FB$ nearly (and for $A = LR, pol$ strictly) independent of QED corrections while $\bar{A}_1^A \approx C_{QED}^A(s)(j^A/r^A - j^T/r^T)\bar{A}_0^A$ reflects effects from the radiative tail and is responsible for deviations of the asymmetry from being constant ($j \neq 0$) and from a straight line ($C_{QED} \neq$ const.); see figure 1 [7].

Any asymmetry at LEP 1 is fully described by two free parameters. The $\mathcal{MI}$ approach allows to determine the contribution of the γZ interference to the asymmetries. It will get more importance when more precise data will be available as is expected in the near future. Maybe it will help to understand the origin of the discrepancy between the LEP 1 measurements and the SLD determination of A_{LR} [16], which corresponds to $\sin^2\vartheta_W^{eff} = 0.229\,4 \pm 0.001\,0$.

Figure 1: *The forward-backward asymmetry for the process $e^+e^- \to \mu^+\mu^-$ near the Z peak.*

4 The $\mathcal{SM}$ approach

The cross section $\sigma_0(s)$ in (2) may be parametrized in the $\mathcal{SM}$ by four electroweak form factors ([8] and references therein): $\rho_{ef}, \kappa_e, \kappa_f, \kappa_{ef}$. To a good approximation, these form factors are independent of s and $\cos\vartheta$ on which they depend in fact, and are even universal (with small deviations) in the sense that they are flavor independent (with the notable exclusion of b quark production in view of the large t quark mass). The effective couplings are related to them as follows: $|a_f^{eff}| = \sqrt{\rho_{ef}}$, $v_f^{eff} = a_f^{eff}(1 - 4|Q_f|s_W^{2,eff})$, $s_W^{2,eff} = \kappa_f \sin^2\vartheta_W$, $v_{ef} \approx v_e^{eff} v_f^{eff}$. For a detailed discussion I refer to [2]. The Z exchange contribution to the cross section and to two of the asymmetries is at the Z peak:

$$
\begin{aligned}
\sigma_0^Z(M_Z^2) &\sim \left(1 + |v_e|^2 + |v_f|^2 + |v_{ef}|^2\right), & (18)\\
\mathcal{A}_{FB}^Z(M_Z^2) &\approx \frac{3}{4}\frac{\Re e\left[v_e v_f^* + v_{ef}\right]}{1 + |v_e|^2 + |v_f|^2 + |v_{ef}|^2} \sim \frac{3}{4}\mathcal{A}_e\mathcal{A}_f, & (19)\\
\mathcal{A}_{LR}^Z(M_Z^2) &= \frac{\Re e\left[v_e + v_{ef} v_f^*\right]}{1 + |v_e|^2 + |v_f|^2 + |v_{ef}|^2} \sim \mathcal{A}_e, & (20)
\end{aligned}
$$

with $\mathcal{A}_f = 2v_f a_f/(v_f^2 + a_f^2)$ and $|a_f^0| = 1$. I is easy to find relations between the parameters of the $\mathcal{MI}$ approach and the parameters of the $\mathcal{SM}$.

As mentioned in the Introduction, one of the unknown parameters in the $\mathcal{SM}$ is the t quark mass. A global fit to the Z line shape data yields [14]:

$$
\begin{aligned}
M_Z &= 91.188\,7 \pm 0.004\,4 \text{ GeV},\\
m_t &= 173 \pm 13 \pm 20 \text{ (H) GeV},\\
\alpha_{strong}(M_Z^2) &= 0.126 \pm 0.005 \pm 0.002 \text{ (H)}, & (21)
\end{aligned}
$$

where 'H' indicates the estimated Higgs boson uncertainty. The corresponding effective weak mixing angle is [1]:

$$\sin^2\vartheta_W^{eff} = 0.232\,2 \pm 0.000\,4 \pm 0.000\,2 \text{ (H)}. \qquad (22)$$

The t quark mass value has to be compared to the result of the direct t quark search at Fermilab with evidence for $m_t = 174 \pm 10 \pm 13$ GeV [15].

All the $\mathcal{SM}$ determinations agree well with the findings of the $\mathcal{MI}$ approach, which are quoted above, and with the evidence of CDF for the t quark. Besides QED and QCD higher order corrections, the fermionic and bosonic weak one loop corrections have to be taken into account. Although leading higher order weak corrections play some role it is not clear whether a complete two loop calculation will be needed finally. The Higgs boson mass cannot be estimated so far.

From the agreement of the $\mathcal{MI}$ and the $\mathcal{SM}$ fits one has to conclude that there are no indications for New Physics presently.

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